A Stylized History of Quantitative Finance[*]

Emanuel Derman

> The evolution of a quantitative approach to finance has proceeded through many small but significant steps and occasional large epiphanies.

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Over the past 70 years financial models have quantified the notion of derivatives, diffusion, risk, diversification, hedging, volatility, replication, and no riskless arbitrage, and explored their consequences.

1. **The Idea of Derivatives, without Diffusion**
   The most productive way of understanding complex things is through their dependence on simpler ones. This idea likely goes back thousands of years, but Spinoza, in his *Ethics* written in the late 17th Century, takes the idea and its consequences seriously. It is a book that could fit in well with modern psychology and behavioral economics.

   Spinoza developed a system of explaining and defining the passions in terms of their relation to three more primitive feelings: **Pain**, **Pleasure**, and **Desire**. Every passion or emotion, in his view, is a derivative of these three sensations, which lie beneath every passion that Spinoza can name. Thus **Hate**, for example, is the **Pain** associated with an external object or person, while **Love** is **Pleasure** accompanied by the idea of an external cause.

   Some passions have two underliers. Envy is **Pain** at someone else's **Pleasure**, analogous to a convertible bond that depends on both equity and interest rates. Similarly, to give a more complex example, Spinoza regards **Cruelty** as the **Desire** to inflict **Pain** on Someone You **Love**, a triple derivatives analogous to a convertible bond that is additionally exposed to credit risk.
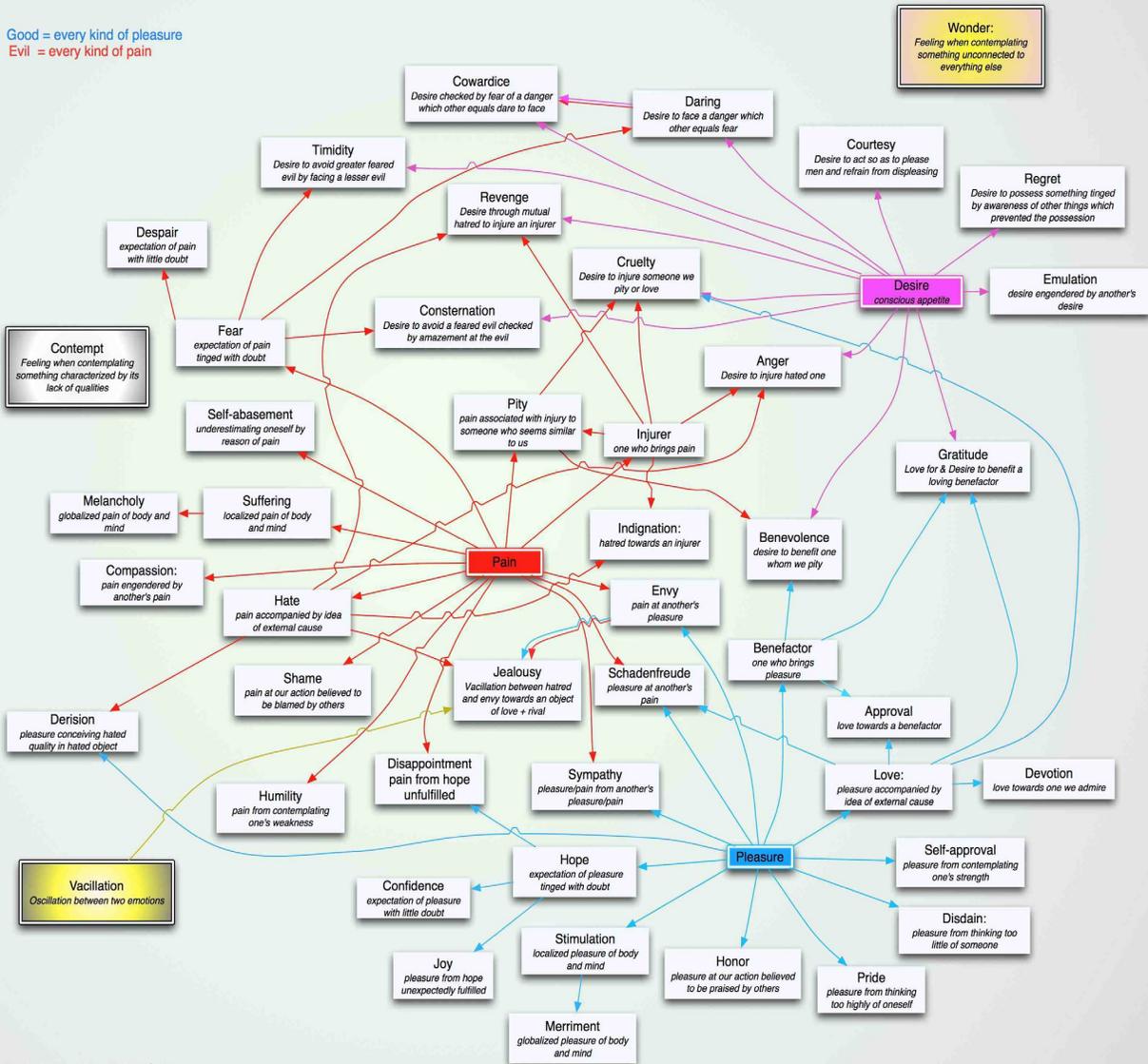
   Spinoza, three hundred years before his time, understood derivatives, but his scheme and definitions are virtually static. I say 'virtually' because, to be fair, Spinoza does allow for fluctuations between passions. He refers to Vacillation, which he defines as the cyclic alternation between two different passions. Thus, for example: **Jealousy** is the **Vacillation** between **Hatred** and **Envy** in relation to a Love object and a rival. So **Vacillation** does describe a kind of volatility – the more rapidly and intensely one **Vacillates**, the greater the **Jealousy**.

Pleasure Pain Desire: A Map of the Emotions *

Emanuel Derman

Good = every kind of pleasure
Evil = every kind of pain

Wonder:
Feeling when contemplating something unconnected to everything else

Cowardice
Desire checked by fear of a danger which other equals dare to face

Daring
Desire to face a danger which other equals fear

Courtesy
Desire to act so as to please men and refrain from displeasing

Timidity
Desire to avoid greater feared evil by facing a lesser evil

Regret
Desire to possess something tinged by awareness of other things which prevented the possession

Revenge
Desire through mutual hatred to injure an injurer

Despair
expectation of pain with little doubt

Cruelty
Desire to injure someone we pity or love

Desire
conscious appetite

Emulation
desire engendered by another's desire

Contempt
Feeling when contemplating something characterized by its lack of qualities

Fear
expectation of pain tinged with doubt

Consternation
Desire to avoid a feared evil checked by amazement at the evil

Anger
Desire to injure hated one

Self-abasement
underestimating oneself by reason of pain

Pity
pain associated with injury to someone who seems similar to us

Injurer
one who brings pain

Gratitude
Love for & Desire to benefit a loving benefactor

Melancholy
globalized pain of body and mind

Suffering
localized pain of body and mind

Indignation:
hatred towards an injurer

Benevolence
desire to benefit one whom we pity

Compassion:
pain engendered by another's pain

Pain

Envy
pain at another's pleasure

Hate
pain accompanied by idea of external cause

Benefactor
one who brings pleasure

Shame
pain at our action believed to be blamed by others

Jealousy
Vacillation between hatred and envy towards an object of love + rival

Schadenfreude
pleasure at another's pain

Derision
pleasure conceiving hated quality in hated object

Approval
love towards a benefactor

Disappointment
pain from hope unfulfilled

Sympathy
pleasure/pain from another's pleasure/pain

Love:
pleasure accompanied by idea of external cause

Devotion
love towards one we admire

Humility
pain from contemplating one's weakness

Self-approval
pleasure from contemplating one's strength

Hope
expectation of pleasure tinged with doubt

Pleasure

Vacillation
Oscillation between two emotions

Confidence
expectation of pleasure with little doubt

Disdain:
pleasure from thinking too little of someone

Stimulation
localized pleasure of body and mind

Honor
pleasure at our action believed to be praised by others

Pride
pleasure from thinking too highly of oneself

Joy
pleasure from hope unexpectedly fulfilled

Merriment
globalized pleasure of body and mind

according to *Ethics*, Benedict de Spinoza

2. **The Idea of Diffusion, but without Derivatives**

    In the 19th Century physicists began to analyze the diffusion of gases.

    Thomas Graham in 1831-33: "...gases of different nature, when brought into contact, do not arrange themselves according to their density, the heaviest undermost, and the lighter uppermost, but they spontaneously diffuse, mutually and equally, through each other, and so remain in the intimate state of mixture for any length of time."

    Wikipedia: "In 1858, Rudolf Clausius introduced the concept of the mean free path (the average distance traveled between collisions that characterize the diffusion). In the same year, James Clerk Maxwell developed the first atomistic theory of transport processes in gases. The modern atomistic theory of diffusion and Brownian motion was developed by Albert Einstein, Marian Smoluchowski and Jean-Baptiste Perrin. Ludwig Boltzmann, in the development of the atomistic backgrounds of the macroscopic transport processes, introduced the Boltzmann equation, which has served mathematics and physics with a source of transport process ideas and concerns for more than 140 years."

    So physicists understood diffusion the particles that comprise gases but never applied the idea to functions (i.e. derivatives) of the diffusing substances. They discussed the behavior of underliers, but not functions of underliers.

    [The exception is of course Bachelier, in this thesis on speculation in France. In the opening year of the 20th Century he developed and applied the theory of diffusion to finance and derived the equations for Brownian motion. He applied them to options with underliers that undergo arithmetic Brownian motion. Far ahead of his time, his work was only rediscovered by the finance community in the 1960s.

3. **The Definition of Risk σ as Standard Deviation of Returns**

    In the 1950s, Markowitz applied statistics to highlight the tension between risk and return, with risk measured as the standard deviation of returns. Assuming a knowledge of the future distribution of returns for each stock, he found the portfolio with the most expected return μ for a given standard deviation σ, or with the least standard deviation for a given expected return.

4. **The Idea of Replication as a Strategy for Valuation.**
    (If you like your leverage you can keep your leverage)

    1958: Modigliani and Miller showed that the value of a firm is independent of its leverage: You can recreate a levered firm for yourself by borrowing money yourself (rather than the firm levering itself) to buy the stock of an unlevered firm. You will then have created the levered firm for yourself, and it still has the same value.

5. **Early 1960: The Capital Asset Pricing Model answers the Key Question of Finance: What return μ should you expect for taking on risk σ ?**

Here is my explanation of how financial theory finds an answer to this question.

The Principle: Equal unavoidable risks of magnitude σ should expect equal returns of magnitude μ.

How can you avoid or mitigate the risk of a security with risk σ ?

   Hedging: Combine the security with an anticorrelated security.

   Diversification: Combine the security with many uncorrelated securities to decrease its risk.

Let's apply these principles successively:

(a) Hedging
Consider a world with an infinite number of stocks S_i with expected return μ_i and risk σ_i, each of which are correlated with the market M with correlation ρ_i. Let the expected return of the market be μ_M with risk σ_M
The let's first hedge away the market risk by creating an infinite number of so-called

"market-neutral stocks" P_i = long one stock S_i and short $\beta_i \dfrac{S_i}{M}$ shares of the market M

where $\beta_i = \rho_i \dfrac{\sigma_i}{\sigma_M}$

(b) Diversification
Now we have an infinite number of uncorrelated market neutral stocks. Combine them into a diversified portfolio. Then the volatility becomes asymptotically zero as their risks cancel, statistically. Thus the portfolio replicates a riskless bond.

(c) Replication
Since this portfolio has eliminated all risk, it replicates a riskless bond. By the Law of One Price, it must earn the riskless return r. The statement that P_i earns the riskless return is precisely the statement

$$(\mu_i - r) = \beta_i(\mu_M - r)$$

the Capital Asset Pricing Model, which can easily be extended to more than one factor M.

6. Finance in One Sentence

Richard Feynman was asked for one sentence to leave to posterity from which our knowledge of physics could be reconstructed if all other knowledge of it were lost. His answer was "Everything is made out of atoms." (I have since heard that there is an equivalent sentence for medicine: "Disease is caused by germs.") Here is my one sentence to recreate all of financial theory:

IF

you can hedge away all correlated risk of a security

AND

you can then diversify over all remaining uncorrelated risks

THEN

you should expect only to earn the riskless rate

7. Why is the Capital Asset Pricing Model bad?

Because risk is not as simple as the standard deviation of returns.
Because the market M and the stock S are not really stably correlated.?
Markets are not exactly like flipping coins. Markets depend on psychology, coins don't. There isn't a well-defined probability of a market crash.

8. Option Models in the 1960s involve Volatility but no Replication

Various people (Samuleson, Sprenkel, Ayres, Boness …) developed models of derivatives that are actuarial in that they define the value of an option as the expected discounted value of its payoffs. This value does of course depend on the volatility of the stock. But they don't know what rate of return to use for growing the stock price into the future, and they don't know what rate to use for discounting the payoffs.

People who wanted to use this model had to forecast the return of the stock and figure out what discount rate to use as a consequence of its risk. It was personal.

9. Black-Scholes and Merton (1971-3): The Combination of Derivatives, Volatility, Hedging and Replication in one Package.

Use diffusion for the underlying stock price
Hedge to eliminate stock risk from option
Require that hedged portfolio, which is riskless, earns the known riskless rate.
Then we get the same formula for the option value as the actuarial one, but where all growth and discount rates are riskless rates.

The value of the option does not depend on the expected return of the stock, since that has been hedged away. Instead it depends on the riskless rate, which is known, and on the future volatility of the stock.

10. Why does option theory work better than CAPM?

Because you really can hedge an option with a stock, because their correlation is really close to 1. So even if you don't believe that risk is the standard deviation of returns, the two securities really are deeply connected, more deeply than two stocks are to each other.

11. 1970s: Using the Black-Scholes Equation
Now, instead of forecasting the return of the stock, traders must forecast the volatility of the stock.

Black and Scholes set about calculating the "correct" value of an option by using historical volatilities to estimate future volatilities. But who knows what future volatility will be?

12. Calibration: The Discovery of Implied Volatility (1976)
Latane and Rendelman suggest fitting option market prices to the Black-Scholes formula and extracting the WISD (weighted implied standard deviation of the stock). They then suggest calculating hedge ratios from the model using the WISD.

But implied volatilities are unstable.

This process must be repeated as the implied volatility keeps changing from moment to moment, so there is *something not quite right*.

Implied volatilities tell you that if you believe the model, then, given an option price, this is what the future must be like. But its estimate of the future behavior of the underlier keeps changing.

Nevertheless, from now on everyone calibrates models.

Most people don't even realize that calibration was an invention.

13. Volatility as an Asset Class
Implied volatility is an estimate of future volatility.

Black-Scholes-Merton say that if you know the future volatility, you can replicate an option, and therefore value it.

But the estimate keeps changing.

So you can't really replicate: instead you can speculate on volatility, using options.

If you use the model, and hedge the option, an option now become a way of trading volatility rather than a way of speculating on the stock.

In the future: Volatility as an asset class; Variance swaps.

14. 1987: The Appearance of the Smile

Until 1987, options on the same stock with the same expiration but with different strikes all had roughly the same implied volatility. Thus, even though the implied volatilities changed every day, at least they all moved up or down together for different strikes.

After the 1987 crash, low-strike index options have higher implied volatilities than higher-strike options.

When you fit the model to different option strikes, each one implies a different future volatility for the underlying stock. But in the model a stock can only have one volatility. N*ow something is really wrong* – the BS model can NOT accommodate different volatilities for the same stock.

Nevertheless, people keep using the model inconsistently to estimate hedge ratios as they fit it to a given option.

15. 1994 - … The Smile

Extensions of the idea of diffusion: local volatility, stochastic volatility, jumps …

More complexity without true knowledge of the parameters
More calibration: we use the market to imply the model's parameters - e.g. the volatility of volatility in a stochastic volatility model.

But markets change and these implied parameters are themselves unstable and random, though there are now more of them.

So now, using the stochastic volatility model as an example, you have a market for trading volatility of volatility.

Practitioners and academics develop consistent extensions of Black-Scholes that attribute one process to the underlying stock, but it's not geometric Brownian motion with a fixed volatility, but rather more complicated processes: e.g. local volatility or stochastic volatility.

16. The Future?
An infinite regress of models?

Each model is inadequate, introduces new parameters, which, as the model is embraced, become random quantities the market can speculate and trade on.

Derivatives are not really derivative, except at expiration.[†]

It takes all of these securities — stocks, options, options on options, volatility, volatility of volatility … to define the possibilities of the market.

[†]. Ayache, *The Medium of Contingency*

The probabilistic approach to distributions is a fallacy. The "probabilities" only come into existence after an event, which is offering a price.

All we have in the market are prices of contingent claims of varying complexity. Probabilistic models and pricing kernels and derivative valuation tools are only internal episodes that we require in order locally and always imperfectly to hedge something. We have to keep in mind … that those episodes are present and useful only insofar as they will be recalibrated.[†]